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Determination of a charged-particle-bunch shape from the coherent far infrared spectrum

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Since perturbed relativistic charged particle bunches of millimeter or submillimeter size emit coherently in the far infrared frequency region, there is growing interest in using this spectrum to obtain precise information about the bunch form factor. It is described here how the maximal information, including bunch asymmetry, can be extracted from such measurements.

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The realization that the energy spread of an appropriately tailored charge distribution can be readily compressed [1] has brought attention to the experimental problem of measuring the shape of single bunches on a subpicosecond time scale. Although it has been recognized for some time that coherent radiation with wavelength comparable to or larger than the bunch length would be generated in synchrotron orbits [2–6], it was not until the successful measurements of Nakazato and others [7–11] that the bunch form factor, i.e., the modulus squared of the Fourier transform of the charge distribution, was obtained from the far infrared spectrum. Another advance was the identification and separation of coherent transition radiation [12–14] from which the same bunch information can be obtained. In either case the bunch shape has been calculated from the spectrum by a cosine Fourier transform so that only a symmetric shape can result. In this paper we demonstrate how additional phase information can be recovered from a coherent far infrared spectrum so that detailed bunch properties, such as its asymmetry, can be identified.

To estimate both the incoherent and coherent intensity contributions to the synchrotron radiation spectrum from a bunch, one sums up over the N electrons the total electric field at frequency ω to obtain the total intensity at the detector [3]:

$$I_{\rm tot}(\omega)=I(\omega)[N+N(N-1)F(\omega)], \tag{1}$$

where $F(\omega)$ is the form factor. The first contribution in Eq. (1) is the intensity of the N independent sources while the second coherent part takes into account the phase relations between the different charged particles. For relativistic charged particles the radiation appears in the forward direction and, for this limit, the form factor simplifies to

$$F(\omega)=\left|\int_0^\infty dz\, S(z)e^{i(\omega/c)z}\right|^2, \tag{2}$$

where $S(z)$ is the normalized longitudinal distribution function of charged particles in the bunch. Thus a measurement of the coherent emission spectrum gives the longitudinal bunch form factor $F(\omega)$ and hence provides information about the longitudinal bunch distribution function $S(z)$ through the transform expression [10]:

$$S(z)=\frac{1}{\pi c}\int_0^\infty d\omega \sqrt{F(\omega)}\cos\left(\frac{\omega z}{c}\right). \tag{3}$$

Note that because this is a cosine transform any information about the bunch asymmetry does not appear in $S(z)$. Furthermore, even for a symmetric bunch, this expression does not produce a unique bunch shape. Equation (3) indicates that the maximum value always occurs at the center; however, if the bunch consists of two symmetrically placed peaks this bunch distribution function should necessarily have a minimum at the center. The reason for this ambiguity is that the phase information is missing.

To obtain the maximal information about the bunch shape from the measured spectral data, we propose the following analytical method. Define

$$\hat{S}(\omega)\equiv\int_0^\infty dz\, S(z)e^{i(\omega/c)z}\equiv\rho(\omega)e^{i\psi(\omega)}, \tag{4}$$

where $\hat{S}(\omega)$ is the complex form factor amplitude so that the form factor

$$F(\omega)=\hat{S}(\omega)\hat{S}^*(\omega)=\rho^2(\omega). \tag{5}$$

A measurement of $F(\omega)$ over the entire frequency interval gives directly the magnitude of the form factor amplitude, $\rho(\omega)$.

According to Eq. (1) the E field for the coherent part of the emission spectrum can be written as

$$\frac{\mathbf{E}_{\rm tot}(\omega)}{\sqrt{N(N-1)}}=\mathbf{E}_{\rm eff}(\omega)=\hat{S}(\omega)\mathbf{E}(\omega), \tag{6}$$

so that the effective E field at the detector is linearly related by $\hat{S}(\omega)$ to the E field produced by an individual particle. The integration in Eq. (4) is only over positive z since the effective E field cannot reach the detector before that of the first particle located at $z=0$, a consequence of causality. It follows that $\hat{S}(\omega)$ can be analytically continued into the upper half complex plane by virtue of the factor $\exp[-z\,{\rm Im}(\hat{\omega}/c)]$, where z and ${\rm Im}(\hat{\omega}/c)$ are positive.

Since any one of the charged particles along the longitudinal z axis in the bunch could have been used as the origin reference point, the effective E field at the detector would then precede the single particle one. A translation of the origin by d can always be used to remove this possibility since it only introduces an additional phase factor $e^{i(\omega/c)d}$ in the form factor amplitude in Eq. (4) and a phase term linear in frequency does not contribute to the bunch shape. Hence we can set $d=0$ without loss of generality. Next, we assume that if $\hat{S}(\hat{\omega})$ has a zero at a finite frequency, say ω_1, then it goes to zero no faster than a power law $(\omega-\omega_1)^\alpha$ and that as $|\hat{\omega}|\to\infty$, $\hat{S}(\hat{\omega})$ also decays with a power law in frequency. These assumptions are sufficient so that Kramers-Kronig relations can be applied to $\hat{S}(\hat{\omega})$. Note the formal similarity between Eq. (4), which involves an integral over space, and the corresponding expression for the complex degree of coherence, which involves an integral of similar form over positive frequencies [15], or with the input-output response function analysis used in optics to obtain the complex reflectivity at an interface, which involves an integral of similar form over positive time [16].

In analogy with these earlier analyses, we write

$$\ln\hat{S}(\omega)=\ln\rho(\omega)+i\psi(\omega). \tag{7}$$

The real and imaginary parts expressed in Eq. (7) are related by a Kramers-Kronig relation so that if $F(\omega)$, hence $\rho(\omega)$, are given at all frequencies then [16]

$$\psi(\omega)=-\frac{2\omega}{\pi}P\int_0^\infty dx\,\frac{\ln[\rho(x)/\rho(\omega)]}{x^2-\omega^2}. \tag{8}$$

With the aid of Eq. (8), the frequency-dependent phase $\psi(\omega)$ can be found to complete the determination of the frequency dependence of the complex form factor amplitude given in Eq. (4). The normalized bunch distribution function can now be obtained from the inverse Fourier transform of Eq. (4), namely,

$$S(z)=\frac{1}{\pi c}\int_0^\infty d\omega\,\rho(\omega)\cos\left[\psi(\omega)-\frac{\omega z}{c}\right]. \tag{9}$$

This complete expression should be contrasted with Eq. (3). The details about the bunch asymmetry are contained in the frequency-dependent phase factor $\psi(\omega)$ in Eq. (9) with the realization that only the phase component nonlinear in ω provides additional information.

There is one remaining ambiguity in the solution for the bunch shape which should be mentioned. Because $F(\omega)=|\hat{S}(\omega)|^2$, both the particle bunch form factor amplitude and its complex conjugate give the same form factor. According to Eq. (4) a specific particle bunch of finite length σ_z where $S(z<0)=S(z>\sigma_z)=0$ gives $\hat{S}(\omega)$ while the same particle bunch flipped front to back, i.e., $S_f(z)=S(\sigma_z-z)$, gives $\hat{S}_f(\omega)$. Replacing z by (σ_z-z) in Eq. (4) gives the necessary connection between the two, namely,

$$\hat{S}_f(\omega)=\hat{S}^*(\omega)e^{i(\omega/c)\sigma_z}. \tag{10}$$

Hence we cannot distinguish the particle bunch shape from the one flipped front to back. The frequency-dependent phase $\psi(\omega)$ that is obtained from Eq. (8) corresponds to one of the bunch shapes; the other would be found from $[-\psi(\omega)+\omega\sigma_z/c]$. Watching the change in the asymmetry as a function of the accelerator parameters would be required to distinguish between these two possibilities.

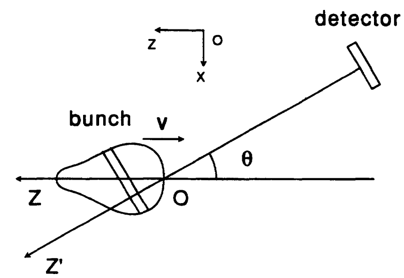

FIG. 1. Schematic view of the axially symmetric bunch with respect to the far ir detector at angle θ. The origin of the coordinate system is located at the front of the bunch. The distance from the source to the detector is much larger than the extent of the bunch. The quantity measured by the far ir spectrum at an angle θ is the projection of the bunch along the z' axis.

Since the experimentally measured spectrum cannot cover the entire frequency interval, asymptotic forms for $F(\omega)$ may be needed in both the low and high frequency regions in order to apply this Kramers-Kronig approach to the data. Because $S(z)$ is necessarily real $\hat{S}(-\omega)=\hat{S}^*(\omega)$ hence $\rho(\omega)$ is an even function of ω, and the Taylor's expansion of $\rho(\omega)$ and $F(\omega)$ at low frequencies must both be parabolic functions. Because the bunch is confined to a finite interval $S(0)=S(\sigma_z)=0$, the expansion of Eq. (4) at high frequencies gives the asymptotic limit $F(\omega)\sim(\omega_0/\omega)^4$, where $\omega_0=c/\sigma_z$.

So far we have focused our attention on a bunch analysis technique whereby the complete longitudinal bunch shape can be obtained from the measured far ir spectrum in the longitudinal direction. However, because of diffraction and formation length features, the far infrared radiation is spread over a much larger angle than the $\theta\sim1/\gamma$ usually associated with radiation from strongly relativistic bunches.

Suppose a measurement of the coherent spectrum is made for radiation at an angle θ with respect to the particle beam axis as shown in Fig. 1. Here we assume an axial symmetric bunch so that $S(z')$ does not depend on the azimuthal angle ϕ, then

$$\hat{S}_\theta(\omega)=\int d\mathbf{r}\,S(\mathbf{r})e^{i(\omega/c)\mathbf{n}\cdot\mathbf{r}}$$

$$=\int dz'\left[\int d\mathbf{r}'_\perp S(\mathbf{r}')\right]e^{i(\omega/c)z'}, \tag{11}$$

where $\mathbf{r}'_\perp$ is perpendicular to the z' axis so that

$$\hat{S}_\theta(\omega)=\int dz' S_\theta(z')e^{i(\omega/c)z'}, \tag{12}$$

and

$$S_\theta(z')=\int d\mathbf{r}'_\perp S(\mathbf{r}'). \tag{13}$$

This "longitudinal" distribution function defined by Eq. (13) is now along the z' axis where the plane perpendicular to the z' axis is defined by the equation $z'=x\sin\theta+z\cos\theta$. For any angle θ, the analysis described earlier goes through as before so that the projection of the bunch along the z' axis is the quantity determined from the far ir spectrum. By measuring this spectrum at a number of angles the longitudinal and transverse size of the bunch can be obtained.

In more general cases where the charged particle bunch possesses still lower symmetry (e.g., a banana shape) then $\hat{S}_{\theta,\phi}(\omega)$ would depend on both θ and ϕ and complete spectrum measurements at different solid angle values would be required to map out the bunch shape.

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